

# ROBUST IMAGE WATERMARKING WITH ZERNIKE MOMENTS

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## Abstract

*In this paper, we studied the image watermarking system based on Zernike moments. Because the magnitudes of Zernike moments have the desirable invariance property against image rotation, we employ Zernike moments as our watermark signal to achieve the robustness against image rotation. The watermark is embedded to the cover image in the spatial domain. Based on whether the image is RST attacked or not, two watermark detection algorithms are proposed and tested. The first algorithm can reconstruct the embedded watermark with the Zernike moments' vector extracted from the testing image if it does not undergo any RST attacks. The second algorithm extracts the feature vector in the format of the magnitudes of the Zernike moments from the testing image and employs the RMSE to detect the watermark.*

**Keywords:** Zernike Moments; Watermarking.

## 1. Introduction

The digital watermark is a signal added to digital contents that can be detected or extracted later. In watermarking applications, robustness is still a challenging issue to be resolved to survive different attacks.

The rotation, scaling and translation (RST) invariant image watermark has been proposed to solve the problem. O'Ruanaidh et al [6] first outlined the theory of integral transform invariants based on Fourier-Mullin transform. Lin et al [5] also gave their algorithm based on Fourier-Mullin transforms. However, one drawback coming with the Fourier-Mullin based methods is the implementation difficulty, which is mainly caused by the interpolation error for implementing the log-polar and inverse log-polar mapping during embedding and detection procedure.

Moments and moments-based image invariants have been used in image recognition extensively [3][8]. Hu [2] first introduced his seven moment-based image invariants that have the invariance property against Affine transformations including rotation, scaling and translation. However, to compute the higher order of Hu's moment invariants is quite complex, and to reconstruct the image from Hu's invariants is

also very difficult. To solve these problems, Teague [7] introduced the concept of Zernike moments to recover the image from moment invariants based on the theory of orthogonal polynomials.

Robust watermarking based on moment-based image invariants has obtained more researchers' attentions in the past few years. The reason for this is that the moments of an image have the invariance property against image rotation, scaling and translation, which is very desirable for robust watermarking. Recently, both Hu's seven moment-based image invariants and Zernike moments have been used in invariant image watermarking. Alghoniemy and Tewfik [1] embed the watermark by modifying the moment values of the image in a way such that a predefined function of the moment invariants lies within a predetermined value. Kim and Lee [4] proposed to apply Zernike moments as the invariant watermark by modifying the normalized Zernike moments vector of the image. Watermark data is added to the cover image in the spatial domain after the reconstruction process. Both algorithms claim the achievements of RST robustness.

## 2. Zernike Moments

Zernike [9] first introduced a set of complex polynomials  $\{V_{nm}(x, y)\}$  which form a complete orthogonal set over the unit disk of  $x^2 + y^2 \leq 1$  in polar coordinates. The form of the polynomials is defined as:

$$V_{nm}(x, y) = V_{nm}(\rho, \theta) = R_{nm}(\rho)e^{im\theta}$$

where  $n$  is positive integer or zero;  $m$  is integers subject to constraints  $n-|m|$  is even, and  $|m| \leq n$ ;  $\rho$  is the length of the vector from the origin to the pixel  $(x, y)$ ;  $\theta$  is the angle between the vector  $\rho$  and  $x$  axis in counterclockwise direction;  $R_{nm}(\rho)$  is Radial polynomial defined as:

$$R_{nm}(\rho) = \sum_{s=0}^{(n-|m|)/2} (-1)^s \frac{(n-s)!}{s! \left(\frac{n+|m|}{2} - s\right)! \left(\frac{n-|m|}{2} - s\right)!} \rho^{n-2s}$$

The Zernike moment of order  $n$  with repetition  $m$  for function  $f(x, y)$  is defined as:

$$A_{nm} = \frac{n+1}{\pi} \iint_{x^2+y^2 \leq 1} f(x, y) V_{nm}^*(x, y) dx dy$$

where  $V_{nm}^*(x, y) = V_{n,-m}(x, y)$ .

To compute the Zernike moment of a digital image, we just need to change the integrals with summations:

$$A_{nm} = \frac{n+1}{\pi} \sum_x \sum_y f(x, y) V_{nm}^*(x, y), x^2 + y^2 \leq 1$$

Suppose we know all Zernike moments  $A_{nm}$  of  $f(x, y)$  up to order  $N$ , we can reconstruct the image by:

$$f'(x, y) = \sum_{n=0}^N \sum_m A_{nm} V_{nm}(x, y)$$

Fig. 1 shows the reconstruction results for image samples of English letters with Zernike moments of order 10, 20 and 40. We see the lower order Zernike moments capture the gross shape information, and the higher order moments capture the fine details of the image.

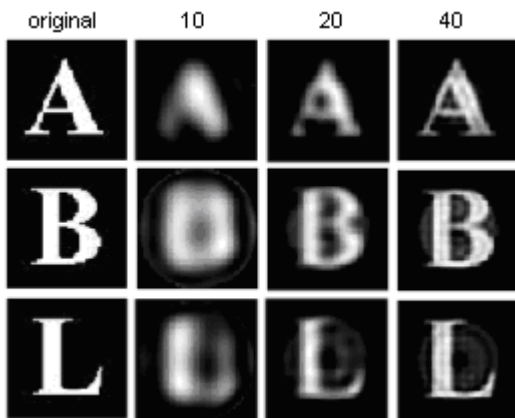


Fig.1. Reconstruction results with Zernike moments of order 10, 20, 40.

The Zernike moments' magnitudes are only invariant to rotation. To achieve scaling and translation invariance, the image needs to be normalized using regular moments [3].

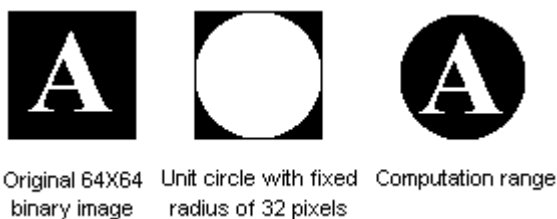


Fig.2. The computation process of the unit disk mapping for Zernike moments.

Zernike polynomials take the unit disk  $x^2+y^2 \leq 1$  as their computation domain. To compute the Zernike moments of a digital image, the range of the image should be mapped to the unit circle with its origin at the image's center. The pixels falling outside the unit circle are discarded in the computation process. For example, if we want to compute Zernike moments of a binary image with spatial resolution of  $64 \times 64$ . This binary image is normalized into a unit circle with fixed radius of 32 pixels. Fig.2 shows the process.

The results of Zernike moments are a series of complex numbers. As discussed already, the magnitudes of Zernike moments are invariant to image rotation. Table 1 compared the magnitudes of the first 10 elements from the Zernike moments series of two  $128 \times 128$  Lena images.

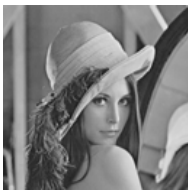

			
Lena.tiff 128x128		90° rotated Lena.tiff 128x128	
Zernike Moment	Magnitude	Zernike Moment	Magnitude
5.0420	5.0420	5.0420	5.0420
0.4936+ 0.2967i	0.5759	0.2967 - 0.4936i	0.5759
0.1753	0.1753	0.1753	0.1753
-0.0010 - 0.4354i	0.4354	0.0010 + 0.4354i	0.4354
-0.0533 - 0.5805i	0.5830	-0.5805 + 0.0533i	0.5830
-0.3400 - 0.2357i	0.4137	0.2357 - 0.3400i	0.4137
0.0869	0.0869	0.0869	0.0869
0.5671 - 0.1504i	0.5867	-0.5671 + 0.1504i	0.5867
0.3562 + 0.0810i	0.3653	0.3562 + 0.0810i	0.3653
0.3518 + 0.1602i	0.3866	0.1602 - 0.3518i	0.3866

Table 1. The comparison of Zernike moments of 2 different Lena images.

### 3. Watermark Embedding and Detection

As we use the Zernike feature vector as the watermark, we try to modify the Zernike moments of the input image. Inserting watermark signal into the Zernike moments of the cover image is the most straightforward method. However, we found that this approach causes severe implementation difficulty. After modifying the Zernike moments of the cover image, we have to reconstruct the watermarked image. Unfortunately, the reconstructed image will not be as same as the original cover image; instead we get a modified version. We reconstructed  $128 \times 128$  gray level Lena images Zernike moments with order up to 40 and we could hardly recognize the reconstructed image. Fig. 3 shows the reconstructed Lena image with Zernike moments up to order 40, and we can see there is a severe fidelity loss after the reconstruction process. Meanwhile, the reconstruction procedure requires Zernike moments with high order and this results in a very high computation load.

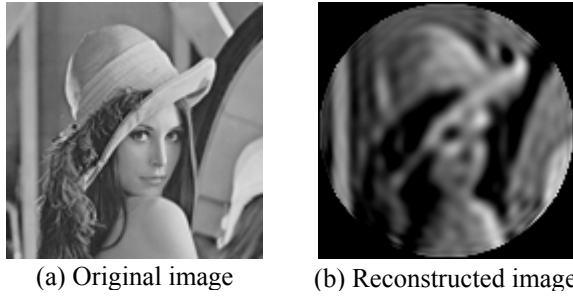


Fig.3. The comparison of original image and reconstructed image with Zernike moments up to order 40.

Because the magnitudes of Zernike moments are invariant against image rotation, we can use them as the watermark. This watermark can be embedded in the spatial intensity domain after reconstruction of the chosen watermark vector consisting of Zernike moments [4]. Embedding strength is controlled by an iterative feature modification and verification procedure. This procedure has a higher accuracy advantage compared to modifying the Zernike moments of the cover image directly.

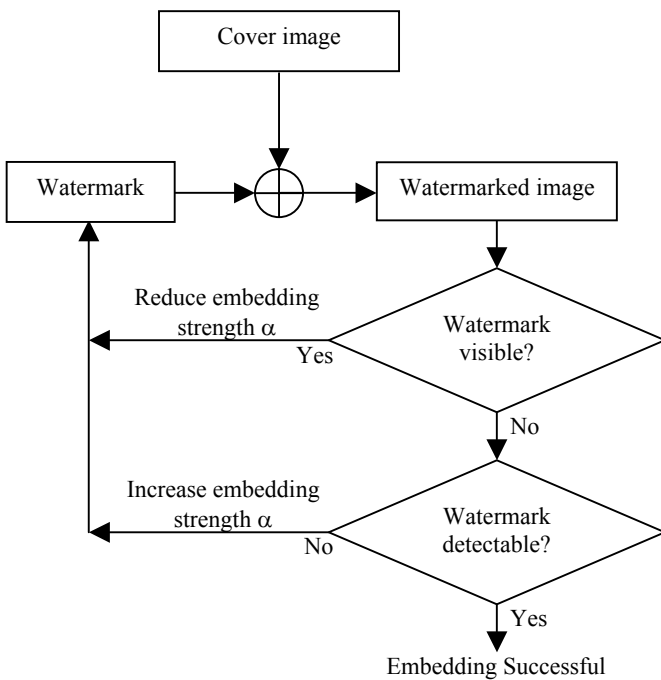


Fig.4. The watermark embedding process.

Fig. 4 shows the whole embedding process. The watermark embedding strength  $\alpha$  is controlled by an iterative feature modification and verification procedure. This procedure avoids the implementation errors that can occur during insertion and detection of the watermark.

At the detector, two detection algorithms are implemented. If the watermarked image does not undergo any RST attacks, we can reconstruct the inserted watermark image with the Zernike moments extracted from the watermarked image. However, if the watermarked image has undergone any RST attacks, we need to rely on the magnitudes of the Zernike

feature vector extracted from the test image. The similarity between magnitudes of the extracted vector and the original watermark vector is computed. We use root-mean-square-error (RMSE) as our similarity measure instead of the traditional normalized correlation because the feature vectors are not in the format of white noise and the correlation measurement cannot produce a peak value when they are in the same vector format. Hence, we employ the distance between two vectors using RMSE function. If the RMSE value is smaller than the threshold, the watermark is detected. The original image is not required at the detector. Fig. 5 illustrates the detection process.

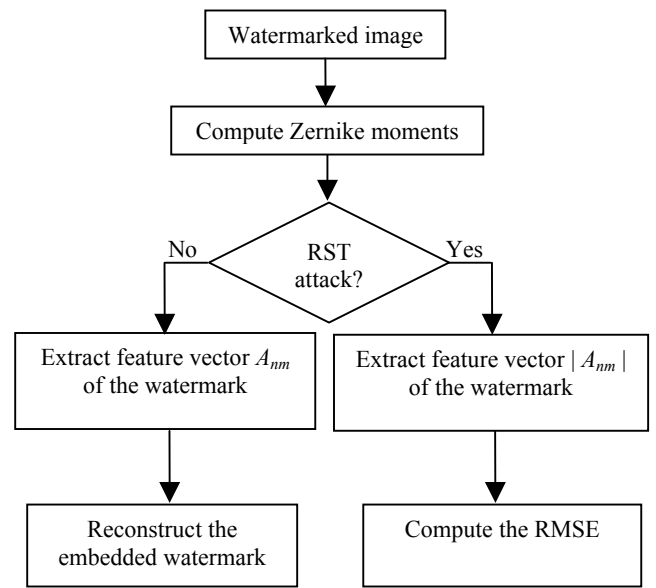


Fig.5. The watermark detection process.

#### 4. Experiment Results

In our experiment, we use a gray level black and white image as our watermark for simplicity. In practical application, the watermark image can be tailored according to the cover image to achieve a more satisfied embedding result with a comparatively higher embedding strength value.

Fig.6 shows the watermark embedding results with different embedding strength  $\alpha$ . This figure shows the iterative embedding process by adjusting the strength  $\alpha$  so that the watermark is not visible in the watermarked image, and meanwhile, still strong enough for us to detect it. The watermarked image achieves satisfied effects when the embedding strength drops to 0.01 for our experiment.

To detect the watermark, the Zernike moments of the test image needs to be computed first. If the image does not undergo any RST attacks, the feature vector  $\vec{p} = (p_1, p_2, p_3, \dots, p_n) = (A_{20}, A_{22}, A_{31}, \dots, A_{n_{\max} m_{\max}})$  can be reconstructed with the Zernike moments of the test image, where  $n$  is the length of the feature vector that is decided by the order of Zernike moments. With the already known Zernike moments' vector of the cover image as the key,

we can extract the embedded Zernike moments of the watermark image. With the same polynomial set  $V_{nm}$  computed from the test image, we can reconstruct the watermark image according to the introduced reconstruction equation. Fig.7 shows the reconstructed watermark image with the Zernike moments up to order 30.

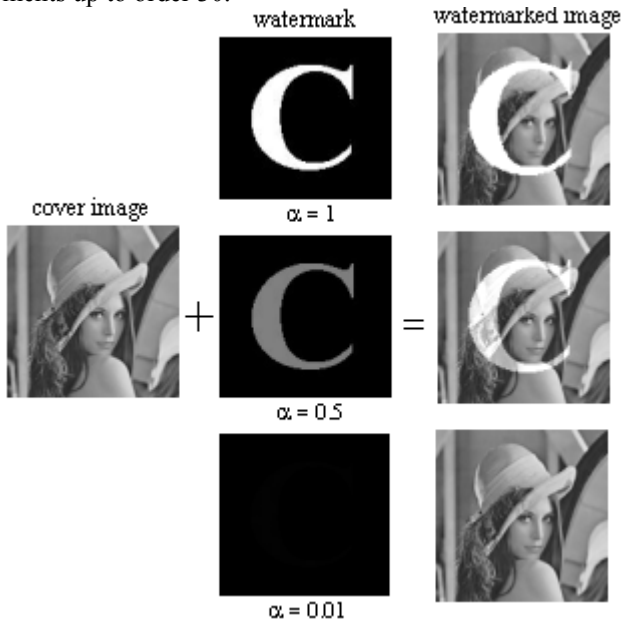


Fig.6. The watermark embedding results with different embedding strengths.



Fig.7. The reconstructed watermark image with extracted Zernike moments up to order 30.

If the image is rotated by a certain degree, we need to employ the magnitudes of the extracted Zernike moments as the feature vector. The similarity  $\bar{s}$  is defined by RMSE between the extracted vector and the watermark as follows:

$$\bar{s}(\bar{p}, \bar{w}) = \frac{1}{N} \left( \sum_{i=1}^N [p_i - w_i]^2 \right)^{1/2}$$

If  $\bar{s}$  is smaller than the detection threshold, the watermark is detected. Otherwise, we claim there is no watermark in the test image. Note that the moments have been modified to known values during the embedding procedure.

We tested a watermarked image rotated for 90 degrees, the computed RMSE value equals  $7.7648 \times 10^{-12}$ . The reason for the small value is that we did not apply any other attack except

rotation, and the Zernike moments' magnitudes have almost same value as the original image. The watermark can be successfully detected by compare the RMSE value with the threshold value set to 0.0001.

## 5. Conclusions and Future Work

In this article, a watermark system based on Zernike moments that provide rotation invariance is studied. The watermark signal is embedded in the spatial intensity domain after reconstruction of the chosen watermark vector consisting of Zernike moments. Embedding strength is controlled by an iterative modification and verification procedure. Two different detection methods are proposed and tested. With the first method, we can successfully reconstruct the embedded watermark image for testing images without RST attacks. The second method employs magnitudes of the Zernike moments and RMSE value to decide whether a watermark is embedded in the RST attacked image. The results show that the watermark based on Zernike moments is robust against image rotation. However, the computation load for high order Zernike moments is costly for the implementation.

For the future work, we need to further test the robustness of this method for scaling and translation attacks. Meanwhile, more efficient watermark embedding algorithm needs to be explored to fully employ the advantages of Zernike moments.

## References

- [1] M. Alghoniemy and A. H. Tewfik, "Geometric Invariance in Image Watermarking," *IEEE Transactions on Image Processing*, Vol.13, No.2, pp.145-153, February 2004.
- [2] M. K. Hu, "Visual pattern recognition by moment invariants," *IRE Trans. Inform. Theory*, Vol.IT-8, pp.179-187, February 1962.
- [3] A. Khotanzad and Y. H. Hong, "Invariant image recognition by Zernike moments," *IEEE Trans. Pattern Anal. Machine Intell.*, vol. 12, pp.489-497, May 1990.
- [4] Hyung Shin Kim and Heung-Kyu Lee, "Invariant Image Watermark using Zernike Moments," *IEEE Transactions on Circuits and Systems for Video Technology*, Vol.13, No.8, pp.766-775, August 2003.
- [5] C. Lin, M. Wu, J. Bloom, I. Cox, M. Miller and Y Lui, "Rotation, scale, and Translation Resilient Watermarking for Images," *IEEE Transactions on Image Processing*, Vol.10, No.5, pp.767-782, 2001.
- [6] J. O'Ruanidh and T. Pun, "Rotation, scale, and translation invariant digital image watermarking," *Signal Processing*, Vol.66, No.3, pp.303-317, 1998.
- [7] M.R.Teague, "Image analysis via the general theory of moments," *J. Opt. Soc. Amer.* 70, pp.920-930, August 1980.
- [8] C. H. Teh and R. T. Chin, "On image analysis by the methods of moments," *IEEE Trans. Pattern Anal. Machine Intell.*, vol. 10, pp. 496-513, April 1988.
- [9] F. Zernike, *Physica*, Vol.1, pp.689, 1934.